Exercise 98

(a) Write $|x| = \sqrt{x^2}$ and use the Chain Rule to show that

$$\frac{d}{dx}|x| = \frac{x}{|x|}$$

(b) If $f(x) = |\sin x|$, find $f'(x)$ and sketch the graphs of f and f'. Where is f not differentiable?

(c) If $g(x) = \sin|x|$, find $g'(x)$ and sketch the graphs of g and g'. Where is g not differentiable?

Solution

Part (a)

Differentiate the given function using the chain rule.

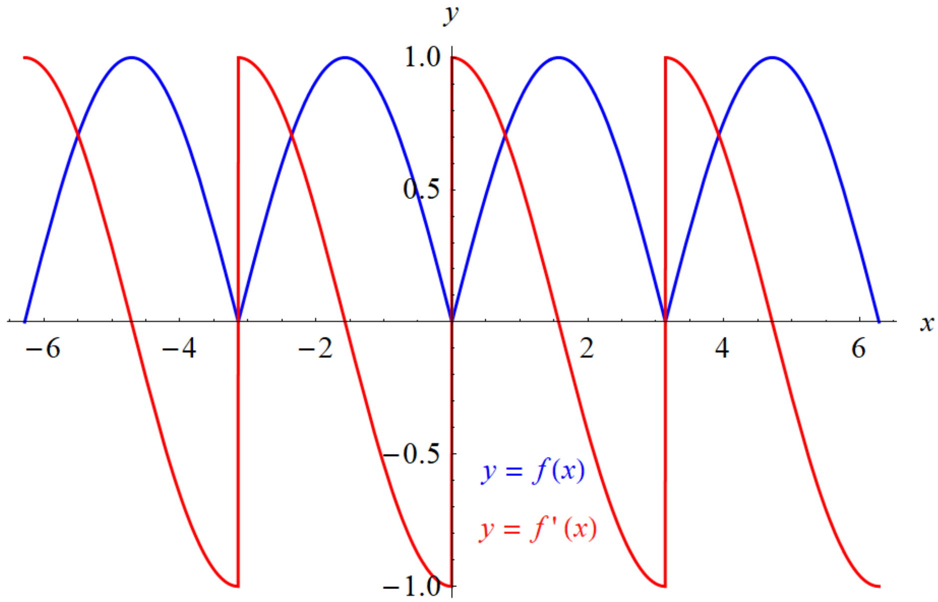
$$\begin{aligned}\frac{d}{dx}|x| &= \frac{d}{dx}\sqrt{x^2} \\ &= \frac{1}{2}(x^2)^{-1/2}\cdot\frac{d}{dx}(x^2) \\ &= \frac{1}{2}(x^2)^{-1/2}\cdot(2x) \\ &= \frac{x}{(x^2)^{1/2}} \\ &= \frac{x}{|x|}\end{aligned}$$

Part (b)

Differentiate the given function $f(x) = |\sin x|$ using the chain rule.

$$\begin{aligned}\frac{d}{dx}|\sin x| &= \frac{d}{dx}\sqrt{\sin^2 x} \\ &= \frac{1}{2}(\sin^2 x)^{-1/2}\cdot\frac{d}{dx}(\sin^2 x) \\ &= \frac{1}{2}(\sin^2 x)^{-1/2}\cdot(2\sin x)\cdot\frac{d}{dx}(\sin x) \\ &= \frac{1}{2}(\sin^2 x)^{-1/2}\cdot(2\sin x)\cdot(\cos x) \\ &= \frac{2\sin x\cos x}{2(\sin^2 x)^{1/2}} \\ &= \frac{\sin 2x}{2|\sin x|}\end{aligned}$$

Below is a graph of $y = f(x)$ and $y = f'(x)$ versus x.



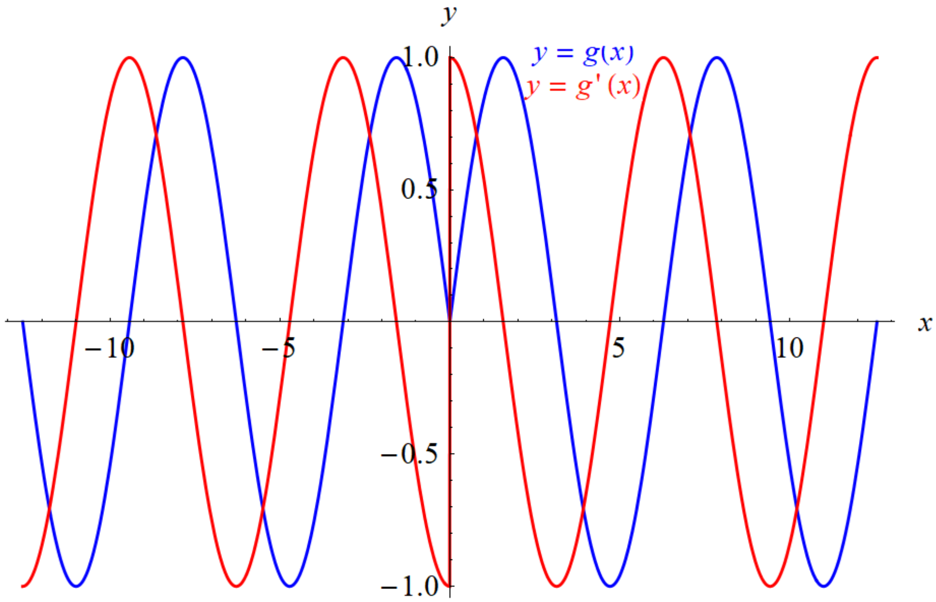
$f(x)$ is not differentiable wherever there are kinks in its graph, that is, $x = n\pi$, where n is an integer.

Part (c)

Differentiate the given function $g(x) = \sin|x|$ using the chain rule.

$$\begin{aligned}
\frac{d}{dx}\sin|x| &= \frac{d}{dx}\sin\sqrt{x^2} \\
&= (\cos\sqrt{x^2})\cdot\frac{d}{dx}(\sqrt{x^2}) \\
&= (\cos\sqrt{x^2})\cdot\left[\frac{1}{2}(x^2)^{-1/2}\cdot\frac{d}{dx}(x^2)\right] \\
&= (\cos\sqrt{x^2})\cdot\left[\frac{1}{2}(x^2)^{-1/2}\cdot(2x)\right] \\
&= (\cos\sqrt{x^2})\cdot\left[\frac{x}{(x^2)^{1/2}}\right] \\
&= \frac{x}{|x|}\cos|x|
\end{aligned}$$

Below is a graph of $y = g(x)$ and $y = g'(x)$ versus x.



$g(x)$ is not differentiable wherever there are kinks in its graph, that is, $x = 0$.